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SETON HALL UNIVERSITY

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Dear Miss Morgan, Refer to: SC-NsG-413/31-002-001

The Final Report on NASA Grant NsG-413/31-002-001 is herein presented. The various research papers written under the grant are briefly reviewed and some conclusions are drawn. I would like to make an observation that may be of interest. Although the grant ended August 31, 1965, I had not completed a paper I had promised on "The General Solution of the Hamiltonian System of Dynamical Equations." The extraordinary difficulties involved has not deterred me from continuing the investigation. I have at present some very encouraging but not fully complete results. The results of this prolonged research will be sent to you on satisfactory completion.

All the papers listed below have been sent to you. The list and a review is given in the following:

I. "The Algebraic Structure of the N-Body Problem" : This was the first in a series of papers that lay the logical and algebraic foundations in the study of the N-Body problem. The group properties of the coefficients of the serial solution of the N-Bodies were considered and some general formulae were established whose utilization was necessary for the sequence of papers that followed. It further specified possible methods in the solution of the N-Body Problem.

II. "The Explicit Solutions of the Analytic N-Body Problem": This paper resolves the problem in explicit serial form with formulae to determine the nth term of the serial coefficients. Use is made of Gamma and Hypergeometric functions to express the coefficients. It further formulates the interval of convergence of the time series for the solution. Though the solution is explicit further research would unfold new types of functions that would prove to be of great utilitarian value. The formulae as they occur may be used for numerical computation of 3, 4,---n-bodies.

III. Revised version of "The Explicit Solutions of the Analytic N-Body Problem": All the formulae and derivations have been re-checked and a new and more potent formulation of the time interval of convergence has been introduced. For further research and numerical work III should be used rather than the older version (II).

IV. "The Regularized Explicit Solution of the Analytic N-Body Problem": This paper deals with the regularized system of differential equations. Though the process parallels that of paper III, it deals more potently with the character of the singularities involved in the solution. Here again further research will unfold types of functions as yet unknown. It may be used for numerical computation on a regularized system of differential equations for the n-body problem.

V. "On an Iterative Process in the Solution of the Regular N-Body Problem": This paper is perhaps the most significant contribution in the series of works on the N-Body problem in the NASA grant project. The methodology no longer is based on the assumption of a converging series as a solution, but applies a combination of an iterative and serial process to lead to a methodology which, as far as I can judge, is a new mathematical process in the solution of a system of differential equations, as applied to the system of equations for the N-Bodies. Not only more potent formulations are generated for the solution, but a method to measure the rate of convergence of a sequence of partial solutions to the actual solution, is formulated.

VI. "On an Arbitrary Function in the Iterative Analytic Solution of a General System of Differential Equations: In a sense this paper is a continuation of the previous paper (IV) but in more general context since it applies to any system of differential equations with certain broad restrictions. In fact I had the principles of this paper in mind when the more concrete paper (V) was written. The developments follow the procedure of the rather new field in analysis, namely Functional Analysis. With the use of integral equations as operational equations, a sequence of approximate solutions are evolved whose limit is the actual solution of the system. An arbitrary function is brought to light which function is the generator of the remaining functions of the sequence. Though the arbitrary function is manifest, it has never been used in the sense it is employed in this paper. The process permits a general formulation of a measure of convergence. This formulation in general is the one that allowed me to recognize the measure of the sequence of functions given in the preceding paper (V).

VII. Revision of the preceding paper (VI) "On an Arbitrary Function in the Iterative Analytic Solution of a General System of Differential Equations. The revision consists of a more concise treatment in the methods of solution by eliminating most of the classical theory of differential equations given in the preceding paper and replacing this theory with the newer notions in Functional Analysis. For further work and study, the revised paper (VII) rather than (VI) should be consulted.

Unfinished Research: as stated at the begining, though the grant ended August 31, 1965, I have continued in my research which I consider to be still part of the NASA project.

VIII. "The General Solution of the Hamiltonian System of Dynamical Equations: Up to this point I have been able to transform the system of Hamiltonian equations to a form amenable to development by a converging time series. A recursive form of a solution has already been attained. But my objective is to get either an irreducible ~~non~~-recursive expression or better still to get a fully explicit solution in terms of the initial conditions only and to determine, if possible, the time interval of the converging time series. This problem has never been solved. Even the recursive form in which the solution has been formulated, is new.

IX. "The Solution of a Type of Linear Diophantine Equation by means of 'Pseudo-Matrices'". This solution has been resolved but its formulation is still unpolished. In the attempt to get a proof for the validity of the general term of paper (III) this particular Diophantine equation appeared. As soon as I have put the result in polished form, I will send it to you, and for possible publication.

Conclusion: The research work carried on by the NASA Grant has accomplished a number of things: (1) It has opened up fields of research in the field of differential equations and specifically so in the N-Body problem which appeared to be closed or nearly so. It has actually solved the N-Body problem by two distinct processes. (2). It has unfolded two types of methodologies in the solution of systems of equations or specialized systems such as the equations of motion of N-Bodies; one of which, as far as can be judged, is distinctly original. Within one of the methodologies (paper VII) the concept of an arbitrary function for an ordinary system of differential equations, has proven to be distinctly useful and the formulation of a rate of measure for convergence significantly useful. (3). The formulae evolved for the N-Body problem should prove meaningful in numerical work in celestial mechanics and in the field of satellite and artificial space bodies. (4). Further work in the direction specified by the papers should lead to discoveries of new functions or enlarge the knowledge of the properties of known functions. (5). The specialization of the general solution to 3-bodies, 4-bodies, etc. should be highly rewarding. (6) Finally the application of the method of paper VII to specialized problems in celestial and analytic mechanics should lead to results which would be otherwise difficult or impossible (from a practial point of view) to achieve.

Sincerely yours,

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